Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813$ mm

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day

1 : answer

(c) $V(t) = 100\pi S(t)$

$V'(7) = 100\pi S'(7) = 602.218$

The volume of water in the can is increasing at a rate of $602.218\ \text{mm}^3/\text{day}$.

$2 : \begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

$2 : \begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

1 : units in (b) or (c)

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

1(a). $\int_0^{60} 2\sin(0.03t)+1.5\, dt$

≈ 171.813 mm

Work for problem 1(b)

$S(60) = \int_0^{60} 2\sin(0.03t)+1.5\, dt$

$= 171.83$ mm.

$\frac{S(60)-S(0)}{60 \text{ days}}$ average rate of change in height.

$\frac{171.813 - 0 \text{ mm}}{60 \text{ days}} \approx 2.864$ mm/day

Do not write beyond this border.

Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 \cdot h$

$\frac{dV}{dt} = \pi \cdot 100 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \pi \cdot 100 \cdot 1.917$

$\frac{dV}{dt} \approx 602.218\ mm^3/day$

At $t=7$ $\frac{dh}{dt} = 2 \cdot \sin(0.03 \cdot 7) + 1.5$

≈ 1.917

Work for problem 1(d)

$M(t) = \frac{3}{400}t^3 - \frac{3}{40}t^2 + \frac{33}{40}t$

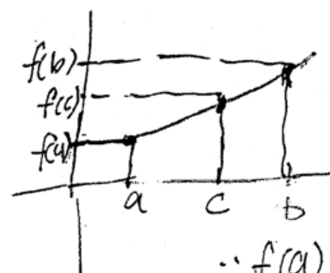
$M'(t) = 3 \cdot \frac{3}{400}t^2 - 2 \cdot \frac{3}{40}t + \frac{33}{40}$

$= \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40}$

$D(t) = \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40} - 2\sin(0.03t) + 1.5$

$D(0) = \frac{33}{40} - 1.5 = -0.675.$

$D(60) = 69.377$



$\because f(a) = -0.675 < 0$

$f(b) = 69.377 > 0$

$\therefore$ There exist a time c which f(c) or $D = 0$ $(M'(t) - S'(t) = 0)$

In order for both the cans' heights to change at the same rate $D(t) = 0 \rightarrow M'(t) - S'(t) = 0$.

$\therefore$ According to the IVT. If a function is continuous on the interval [a,b], and there exist corresponding value f(a) & f(b), in which f(a) < f(b), the c, a value in between (a,b) on the interval [a,b], has a corresponding value in between f(a) & f(b).

Do not write beyond this border.

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Do not write beyond this border.

Work for problem 1(a)

$$\int_0^{60} S'(t)\,dt$$

$$= \int_0^{60} 2\sin(0.03t) + 1.5\,dt$$

$$= 171.813 \text{ millimeters}$$

Work for problem 1(b)

$$\frac{1}{60-0}\int_0^{60} S'(t)\,dt$$

$$= \frac{1}{60}\int_0^{60} 2\sin(0.03t) + 1.5\,dt$$

$$= 2.86356 \text{ mill/day}$$

Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$= 100\pi\left(2\sin(0.03\cdot 7)+1.5\right)$

$= 602 \text{ mil}^3/\text{day}$ at $t=7$

Work for problem 1(d)

$M'(t) = \frac{1}{400}\left(9t^2 - 60t + 330\right)$

$D(t) = \left(\frac{1}{400}(9t^2-60t+330)\right) -$ $: 2\sin(0.03t)+1.5$

$M'(t) = S'(t)$ at $t =$ 11.8166

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

$\int_0^{60} S'(t)\,dt = \int_0^{60} (2\sin(0.03t)+1.5)\,dt = 171.183$ millimeters

Work for problem 1(b)

$S(t) = \int S'(t)\,dt = 1.5x - 66.67\cos(0.03t)+c$

$S(0)=0$

$c = 66.67$, $0 = 1.5(0) - 66.67\cos(0.03(0)) + c$

$S(t) = 1.5x - 66.67\cos(0.03t) + 66.67$

average rate of change =

$\dfrac{S(60)-S(0)}{60} = \dfrac{171.818 - 0}{60} = 28.636$ mm/day

Do not write beyond this border.

Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h$

$V = \pi (10)^2 h$

$V = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$S'(7) = \frac{dh}{dt}\Big|_{t=7}$

$S'(7) = 1.917$ mm/day

$\frac{dV}{dt}\Big|_{t=7} = 100\pi(1.917)$

$= 602.243$ mm^3/day

Do not write beyond this border.

Work for problem 1(d)

$M'(t) = \frac{9x^2}{400} - \frac{3x}{20} + \frac{33}{40}$

$\int_0^{60} D(t)\,dt = \int_0^{60} M'(t) - S'(t)\;dt$

$\int_0^{60} D(t)dt = (60-0)\,D(c)$

$D(c) = 20.4614$

GO ON TO THE NEXT PAGE.

Question 1

Sample: 1A
Score: 9

The student earned all 9 points. In part (d) the student considers $D(0)$ and $D(60)$, notes that they have opposite signs, implies that D is continuous, and invokes the Intermediate Value Theorem to conclude that $D(t)$ must equal 0 for some t in the interval.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 1 point in part (c), no points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student earned the first point with the substitution for $S'(7)$ in the expression for $\frac{dV}{dt}$. Prior to that step, the student was working with $\frac{dh}{dt}$ rather than $S'(t)$. The student's answer is not presented accurately to three decimal places. In part (d) the student's work is incorrect.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no point in part (b), 1 point in part (c), no points in part (d), and the units point. In part (a) the student has the correct limits and integrand but presents an incorrect answer of 171.183 and so earned 2 of the 3 points. In part (b) the student's decimal point is incorrectly placed. In part (c) the student establishes the relationship between V and S by connecting $\frac{dV}{dt}$ to $\frac{dh}{dt}$ and $\frac{dh}{dt}$ to S'. The student uses the truncated value 1.917 for $S'(7)$ in the computation of $\frac{dV}{dt}$, so the student's answer is incorrect. In part (d) the student's work is incorrect.

Question 2

A 12,000-liter tank of water is filled to capacity. At time $t = 0,$ water begins to drain out of the tank at a rate modeled by $r(t),$ measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

(a) $\lim_{t \to 5^-} r(t) = \lim_{t \to 5^-} \left(\dfrac{600t}{t+3} \right) = 375 = r(5)$

$\lim_{t \to 5^+} r(t) = \lim_{t \to 5^+} \left(1000e^{-0.2t} \right) = 367.879$

Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

2 : conclusion with analysis

(b) $\dfrac{1}{8}\int_0^8 r(t)\,dt = \dfrac{1}{8}\left(\int_0^5 \dfrac{600t}{t+3}\,dt + \int_5^8 1000e^{-0.2t}\,dt \right)$

$= 258.052$ or 258.053

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(c) $r'(3) = 50$

The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 $\text{liters}/\text{hour}^2$.

$2 : \begin{cases} 1 : r'(3) \\ 1 : \text{meaning of } r'(3) \end{cases}$

(d) $12{,}000 - \int_0^A r(t)\,dt = 9000$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

Do not write beyond this border.

Work for problem 2(a)

$r(5) = \frac{600t}{t+3} = 375$

$\lim_{x \to 5} 1000e^{-0.2t} = 1000e^{-0.2 \times 5} = 367.879$

no.

Work for problem 2(b)

$\left(\int_0^5 \left(\frac{600t}{t+3}\right) dt + \int_5^8 (1000e^{-0.2t})\, dt \right) \div 8$

$= (1234.507 + 829.915) \div 8$

$= 258.053$ liters/hour.

Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(c)

$$r'(3) = \frac{d}{dt}\left(\frac{600t}{t+3}\right)$$

$$= \frac{(t+3)(600) - (600t)(1)}{(t+3)^2}$$

$$= \frac{3600 - 1800}{36}$$

$$= 50$$

The rate of at which water is draining is increasing at 50 liters/h^2 at $t=3$.

Work for problem 2(d)

$$9000 = 12000 - \int_0^x r(t)\,dt$$

Do not write beyond this border.

Do not write beyond this border.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 2(a)

function is continuous only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

$\lim_{x \to 5^-} \frac{600t}{t+3} = \left\{ \frac{600 \cdot 5}{8} \right\} = 375$

$\lim_{x \to 5^+} 1000e^{-0.2 \cdot 5} = 367.879$

$\lim_{x \to 5^-} f(t) \neq \lim_{x \to 5^+} r(t) \Rightarrow$ function is not continuous a $t = 5$

Work for problem 2(b)

Average rate : $\frac{1}{b-a} \int_a^b f(x)dx$

$\frac{1}{8-0}\left[\int_0^5 \frac{600t}{t+3} dt + \int_5^8 1000e^{-0.2t} dt\right] = 258.0257$ liters per hour

Continue problem 2 on page 7.

Work for problem 2(c)

$$r'(t)\Big|_{0\le t\le 5} = \frac{600(t+3) - 600t}{(t+3)^2}$$

$$r'(t) = \frac{600t + 1800 - 600t}{t^2 + 6t + 9} = \frac{1800}{9+18+9} =$$

$$= \frac{1800}{36} = 50 \ \frac{lit}{hour^2}$$

the rate of change of water at time period $t=3$

Do not write beyond this border.

Do not write beyond this border.

Work for problem 2(d)

$$\int_0^A r(t)\,dt = 9000$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 2(a)

r will be continuous if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$, in our case

$$\lim_{t \to 5^-} r(t) = \lim_{t \to 5^+} r(t)$$

$$\lim_{t \to 5^-} \frac{600t}{t+3} = 375$$

$$\lim_{t \to 5^+} 1000e^{-0,2t} = 367,8794$$

$\lim_{t \to 5^-} \neq \lim_{t \to 5^+}$, so r is discontinuous.

Work for problem 2(b)

$$\int_0^5 \frac{600t}{t+3} + \int_5^8 1000e^{-0,2t} =$$

Continue problem 2 on page 7.

Work for problem 2(c)

$0 \le 3 \le 5$, so we use $r(t) = \frac{600t}{t+3}$

$r'(t) = \frac{600(t+3) - 600t}{(t+3)^2}$

$r'(3) = \frac{600 \cdot 6 - 600 \cdot 3}{6^2} = 50$ liters in quadrant / hour

(liters^2 / hour)

$r(t)$ is a rate

$r'(t)$ is acceleration

Answer: $r'(3) = 50$ liters^2/hour

Work for problem 2(d)

$1000 e^{-0,2t} = 9000$

$e^{-0,2t} = 9$

$-0,2t = \ln 9$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

Do not write beyond this border.

Question 2

Sample: 2A
Score: 9

The student earned all 9 points. In part (a) it is sufficient to show that $r(5) \neq \lim_{t \to 5^+} r(t)$. In part (d) an ideal solution would present an A as the upper limit of integration.

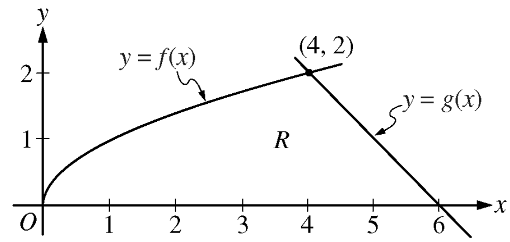
Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student's work is correct. Note that the last digit in the student's calculation looks like an 8, but when compared with the student's work in part (c), this digit is clearly intended to be a 9. In part (b) the student earned the points for the integrand and the limits and constant. The student's answer is not accurate to three decimal places, so the answer point was not earned. In part (c) the student computes the value of $r'(3)$. The student's interpretation of the meaning is vague. In part (d) the student earned the integral point.

Sample: 2C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the integrand point. The response is missing the $\frac{1}{8}$, so no other points were earned in part (b). In part (c) the student computes the value of $r'(3)$. The student's interpretation of the meaning is insufficient. In part (d) the student's work is incorrect.

Question 3



The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$
$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

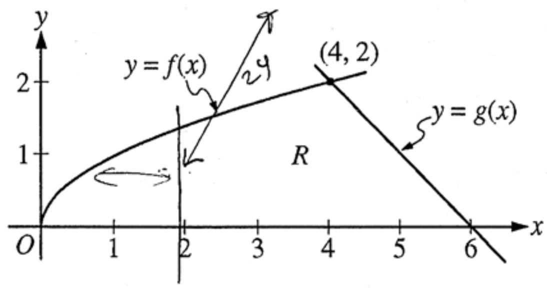
The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

$$R = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx =$$

$$= \frac{2}{3}x^{3/2}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6 = \frac{16}{3} + 36 - 18 - 24 + 8 =$$

$$= 2 + \frac{16}{3} = \frac{22}{3}$$

Do not write beyond this border.

Do not write beyond this border.

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b) ~~Vq~~ for $0 \le y \le 2$: ~~R is between y=f(x) & the horizontal axis;~~

$y = \sqrt{x} \Leftrightarrow x = y^2$; $\begin{cases} x=0 \\ y=0 \end{cases}$; $\begin{cases} x=4 \\ y=2 \end{cases}$;

$\cancel{V = \int_0^2}$ $y = 6-x \Leftrightarrow x = 6-y \Rightarrow$

$$V = \int_0^2 \left((6-y-y^2) \times 2y\right) dy$$

Do not write beyond this border.

Do not write beyond this border.

Work for problem 3(c) tangent line to f;

$$y_t = f(x_0) + f'(x_0)(x - x_0)$$

$$y_t = \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}(x - x_0)$$, y_t is $\perp$ to $g(x) \Rightarrow$

$$\frac{1}{2\sqrt{x_0}} = -\frac{1}{g'(x_0)}$$, $g'(x_0) = (6-x_0)' = -1 \Rightarrow$

$$\frac{1}{2\sqrt{x_0}} = 1 \Leftrightarrow 2\sqrt{x_0} = 1$$

$$\sqrt{x_0} = \frac{1}{2}$$

$\underline{x_0 = \frac{1}{4}}$, $y_0 = \sqrt{x_0} = \underline{\frac{1}{2}} \Rightarrow$

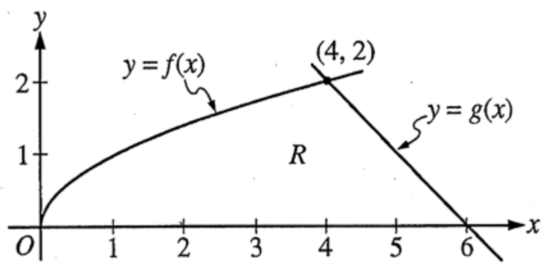
$P(0.25, 0.5)$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

a) $A = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= \frac{2}{3}x^{3/2}\Big]_0^4 + \left(6x - \frac{x^2}{2}\right)\Big]_4^6 = \frac{2}{3}(4)^{3/2} + \left[\left((6\times 6) - \frac{36}{2}\right) - (24-8)\right]$

$= \frac{2}{3}(2)(4) + (18-16) = \frac{16}{3} + 2 = \frac{16}{3} + \frac{6}{3} = \frac{22}{3}(\text{unit})^2.$

Do not write beyond this border.

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

$h = 2y$

$y = 6 - x \Rightarrow x = 6 - y$

$y = \sqrt{x} \Rightarrow x = y^2$

$V = \int_a^b A(x)\,dx = \int_0^2 ([(6-y) - y^2] \times 2[(6-y) - y^2])\,dy$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad g'(x) = -1$

$\frac{1}{2\sqrt{x}} = 1$ $\quad 1 = 2\sqrt{x}$

$\sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

$f(1/4) = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$

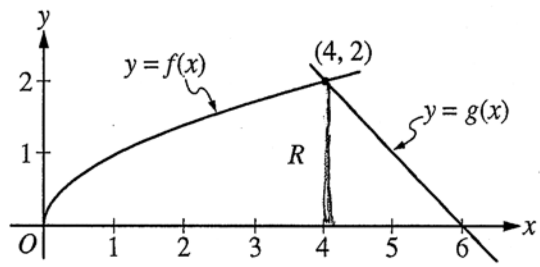
Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Do not write beyond this border.

Work for problem 3(a)

(a). $R = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= 2x^{\frac{3}{2}}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6$

$= 2 \times 4^{\frac{3}{2}} - 0 + 6\times 6 - \frac{36}{2} - 6\times 4 + \frac{16}{2}$

$= 16 + 36 - 18 - 24 + 8$

$= 8$

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

$V = \int_0^6 (6 - x - \sqrt{x})\,dy \quad (0 \le y \le 2)$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}} \qquad g'(x) = -1$

Since the line is perpendicular to $g(x)$, when $g'(x) = -1$.

the slope of the line is 1

Therefore $\frac{1}{2\sqrt{x}} = 1 \qquad x = \frac{1}{4}$

So $f(\frac{1}{4}) = \frac{1}{2}$ the coordinates of point P is $(\frac{1}{4}, \frac{1}{2})$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 3 points in part (a), no points in part (b), and 3 points in part (c). In parts (a) and (c) the student's work is correct. In part (b) the student's integrand was not eligible for any points.

Sample: 3C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), and 3 points in part (c). In part (a) the student earned the integral point. The student's antidifferentiation is incorrect. The student was eligible for the answer point, but the work contains an arithmetic error. In part (b) the student's integrand is incorrect, so the work was not eligible for any points. In part (c) the student's work is correct.

Question 4

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

(c) Given that $f(1) = 2$, determine the function f.

(a) $f'(x) = 0$ at $x = 4$

$f'(x) > 0$ for $0 < x < 4$

$f'(x) < 0$ for $x > 4$

Therefore f has a relative maximum at $x = 4$.

$$3 : \begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$$

(b)
$$\begin{aligned} f''(x) &= -x^{-3} + (4 - x)\left(-3x^{-4}\right) \\ &= -x^{-3} - 12x^{-4} + 3x^{-3} \\ &= 2x^{-4}(x - 6) \\ &= \frac{2(x-6)}{x^4} \end{aligned}$$

$f''(x) < 0$ for $0 < x < 6$

The graph of f is concave down on the interval $0 < x < 6$.

$$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$$

(c)
$$\begin{aligned} f(x) &= 2 + \int_1^x \left(4t^{-3} - t^{-2}\right) dt \\ &= 2 + \left[-2t^{-2} + t^{-1}\right]_{t=1}^{t=x} \\ &= 3 - 2x^{-2} + x^{-1} \end{aligned}$$

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

NO CALCULATOR ALLOWED

Work for problem 4(a)

$f(x) > 0$

for $f'(x) = 0$, $(4-x)x^{-3} = 0 \Rightarrow \frac{1}{x^3} \cdot (4-x) = 0$, $x \neq 0$

$\Rightarrow x = 4$ This is the x-coordinate of the critical point

Derivative test:

$f'(x)$ before $x=4$ is positive

$f'(x)$ after $x=4$ is negative

$f'(x) > 0$ for $0 < x < 4$

$f'(x) < 0$ for $4 < x < +\infty$

Thus the point with the x-coordinate $x = 4$ is a relative maximum since at that point $f(x)$ goes from increasing to decreasing.

Work for problem 4(b)

$f'(x) = 4x^{-3} - x^{-2}$

f is concave down for $f''(x) = 0$

$f''(x) = -12x^{-4} + 2x^{-3} = 0$

$\Rightarrow -6x^{-4} + x^{-3} = 0$

$\Rightarrow x^{-3}\left(\frac{-6}{x} + 1\right) = 0$

$\Rightarrow \frac{1}{x^3}\left(-\frac{6}{x} + 1\right) = 0 \Rightarrow 1 - \frac{6}{x} = 0$

$\Rightarrow x = 6$

$f''(x) < 0$ for $0 < x < 6$

and $f''(x) > 0$ for $6 < x < +\infty$

Thus the interval on which f is concave down is $0 < x < 6$

Do not write beyond this border.

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$$f(x) = \int_1^x (4u^{-3} - u^{-2})\, dx + 2$$

$$= \left[-2u^{-2} + u^{-1}\right]_1^x + 2$$

$$= -2x^{-2} + x^{-1} - [-2+1] + 2$$

$$= -2x^{-2} + x^{-1} + 2 - 1 + 2$$

$$= -2x^{-2} + x^{-1} + 3.$$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 4(a)

$f'(x) = (4-x)x^{-3} = 0$

$4x^{-3} - x^{-2} = 0$

$x^{-2}\left(\frac{4}{x} - 1\right) = 0$

$x = 0, \quad \frac{4}{x} - 1 = 0$

$\frac{4}{x} = 1$

$x = 4$

interval	$x < 0$	$0 < x \le 4$	$x > 4$
sign $f'(x)$	$-$	$+$	$-$
Behavior	Decr	Incr	Decr

at $x = 0 \rightarrow$ relative minimum

at $x = 4 \rightarrow$ relative maximum

Work for problem 4(b)

$f''(x) = (4-x)(-3x^{-4}) + (x^{-3})(-1)$

$= -12x^{-4} + 3x^{-3} - x^{-3}$

$= -x^{-3}(12x^{-1} - 3 + 1) = 0$

$\frac{12}{x} = 2$

$x = 6$

Interval	$x < 6$	$x \ge 6$
sign $f''(x)$	$+$	$+$
Behavior	concave up	concave up

f is never concave down because $f''(x)$ is always positive

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$f(x) = \int 4x^{-3} - x^{-2}\, dx$

$= -2x^{-2} + x^{-1} + C$

$2 = -2(1)^{-2} + (1)^{-1} + C$

$2 = -2 + 1 + C$

$C = 3$

$\downarrow$

$f(x) = -2x^{-2} + x^{-1} + 3$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 4(a)

$f'(x) = 0$

$(4-x)\,x^{-3} = 0$

$x^{-3} = 0 \quad | \quad x = 4$

$x = 0$

$f'(4) = (4-4)(4)^{-3} = 0$

~~neither~~

we cant tell if this is a relative max or min, so its neither

Work for problem 4(b)

$u = 4-x \quad | \quad v = x^{-3}$

$u' = -1 \quad | \quad v' = -3x^{-4}$

$f''(x) = -x^{-3} + (-12x^{-4} + 3x^{-3})$

x	$(0,4)$	$(4,\infty)$
$f''(x)$		

at $(0,4)$ its a concave down

Do not write beyond this border.

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$u = \cancel{4}-x \qquad dx = -du$

$du = -1\ dx$

$f(x) = \int \cancel{x}(4-x)x^{-3}\ dx$

$= -\left[\ \frac{ux^{-2}}{-2}\ \cancel{\$}+C\right]$

$= \frac{(4-x)\ x^{-2}}{2} + C$

$f(1) = \frac{(4-1)\ (1)^{-2}}{2} + C$

$2 = \frac{3}{2} + C$

$\boxed{C = \frac{1}{2}}$

$f(x) = \frac{(4-x)x^{-2}}{2} + \frac{1}{2}$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

Question 4

Sample: 4A
Score: 9

The student earned all 9 points. In part (c) the student's differential does not match the variable in the integrand, but no points were deducted.

Sample: 4B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c). The student was eligible for 1 of the 3 points in part (a), which was earned by showing that f has a relative maximum at $x = 4$. In part (b) the student correctly computes $f''(x)$, so the first 2 points were earned. In part (c) the student's work is correct.

Sample: 4C
Score: 3

The student earned 3 points: no points in part (a), 2 points in part (b), and 1 point in part (c). In part (a) the student incorrectly declares that both $x = 0$ and $x = 4$ are x-coordinates of critical points. In part (b) the student correctly computes $f''(x)$, so the first 2 points were earned. The student's interval is incorrect, so the answer point was not earned. In part (c) the student presents a differential equation solution. For a correct solution, the first point was earned for a correct antiderivative, the second point for use of the initial condition, and the third point for the answer. This student's antiderivative is incorrect, so the first point was not earned. The initial condition is used correctly, so the second point was earned. Because the student's antidifferentiation is incorrect, the student was not eligible for the answer point.

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

(b) $\int_0^{60} |v(t)|\,dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$.

$\int_0^{60} |v(t)|\,dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139$ meters

2 : { 1 : meaning of integral; 1 : approximation }

(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

2 : { 1 : difference quotient; 1 : conclusion with justification }

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$L'(40) = \dfrac{B(40)v(40)}{L(40)} = \dfrac{9 \cdot 2.5}{\sqrt{144 + 81}} = \dfrac{3}{2}$ meters/sec

3 : { 1 : derivatives; 1 : uses $B'(t) = v(t)$; 1 : answer }

1 : units in (a) or (b)

NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Work for problem 5(a)

$a(5) = \frac{v(10) - v(0)}{10 - 0} = \frac{2,3 - 2,0}{10} = 0,03$ meters/(second)^2

Work for problem 5(b)

$\int_0^{60} |v(t)|dt$ means a total distance, travelled by Ben during the time = 60 seconds, $0 < t < 60$.

$\int_0^{60} |v(t)|dt = v(0) \cdot \Delta t + v(10) \Delta t + v(40) \Delta t =$

$= 2 \cdot 10 + 2,3 \cdot 30 + 2,5 \cdot 20 = 139$ meters

Do not write beyond this border.

Continue problem 5 on page 13.

Work for problem 5(c)

Accoarding tc Mean Value Teozem

$$\frac{B(60)-B(40)}{60-40} = \frac{49-9}{20} = 2 \text{ ;}$$

$\Rightarrow$ There must be a time t when velecity is equal to 2 meters/second

Work for problem 5(d)

$$I_S \quad (L(t))^2 = 12^2 + (B(t))^2$$

$$L(t) = \sqrt{12^2 + (B(t))^2}$$

$$L'(t) = \frac{2B(t) \cdot B'(t)}{2\sqrt{12^2 + (B(t))^2}}$$

$$L'(40) = \frac{9 \cdot 2{,}5}{\sqrt{225}} = \frac{9 \cdot 2{,}5}{15} = \frac{3 \cdot 2{,}5}{5} = \frac{7{,}5}{5} =$$

$$= 1{,}5 \text{ meters/second}$$

Do not write beyond this border.

Do not write beyond this border.

NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Work for problem 5(a)

$a(5) = v'(5) = \frac{v(10)-v(0)}{10-0} = \frac{2.3-2}{10} = 0.03$

Work for problem 5(b)

the meaning of $\int_0^{60} |v(t)|\,dt$: The total distance Ben rides from $t=0$ to $t=60$, which is measured by meters.

Left Riemann Sum approximation of $\int_0^{60} |v(t)|\,dt$:

$\int_0^{60} |v(t)|\,dt = 2\times10 + 2.3\times(40-10) + 2.5\times(60-40)$
$= 20 + 69 + 50$
$= 139$

Do not write beyond this border.

Continue problem 5 on page 13.

NO CALLOUT ALLOWED

Work for problem 5(c)

uncertain. the total change from $t=40$ to $t=60$ is increase. If the velocity is 2 meters per second, there will be a decrease in $40 \le t \le 60$.

We can't find the exact change between $40 \le t \le 60$, so there is uncertainly a velocity is 2 meters per second

Work for problem 5(d)

$L(t)^2 = 12^2 + B(t)^2$ when $t=40$

$2L(40)\frac{dL}{dt} = 144 + 2B(40)\frac{dB}{dt}$ and $L(40) = \sqrt{144+B(40)^2}$

$= \sqrt{144+81}$

$2\times15\frac{dL}{dt} = 144 + 2\times9\frac{dB}{dt}$

$= \sqrt{225}$

$= 15$

$\frac{dL}{dt} = \frac{144+45}{30}$

$= 63$ meters per second

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Work for problem 5(a)

$a(t)$ at $t=5$: $\dfrac{v(10)-v(0)}{10-0}$

$= \dfrac{2.3-2.0}{10}$

$= \dfrac{.3}{10}$

$\boxed{= .03 \text{ m}^2/\text{second}}$

Work for problem 5(b)

$\int_0^{60} |v(t)|\,dt$ would be the total distance covered by Ben, whether going forwards or backwards.

$$\begin{array}{r} 136 \\ \times\ 30 \\ \hline 4080 \end{array}$$

$\int_0^{60} |v(t)|\,dt \approx 10(100)+30(136)+20(9)$

$= 1000 + 4080 + 180$

$= \boxed{5260 \text{ meters}}$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$= \frac{49-9}{60-40}$

$= \frac{40}{20}$

$= \boxed{2\ m/s}$

According to the mean value theorem, there is a time $t=c$ in which Ben's velocity is 2 meters per second.

Work for problem 5(d)

$(L(t))^2 = 12^2 + (B(t))^2$

$(L(t))^2 = 144 + (B(40))^2$

$(L(t))^2 = 144 + 9^2$

$\sqrt{(L(t))^2} = \sqrt{225}$

$L(t) = 15$

$\frac{d}{dx}$

$$\begin{array}{r} 144 \\ +\ 81 \\ \hline 225 \end{array}$$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

Question 5

Sample: 5A
Score: 9

The student earned all 9 points. In part (d) the student solves for $L(t)$ prior to differentiating.

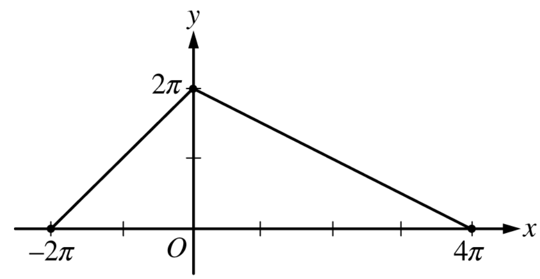
Sample: 5B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), no points in part (c), 2 points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student's work is incorrect. In part (d) the student differentiates the expression incorrectly. The student uses $B'(40) = v(40) = 2.5,$ so the second point was earned. Because the student's derivative is of the form $LL' = BB' + C,$ where C is a constant, the student was eligible for the answer point. The student's answer is consistent with the expression presented, so the answer point was earned. The student earned the units point, because "meters" are mentioned in the interpretation of the meaning of the integral in part (b).

Sample: 5C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), 2 points in part (c), no points in part (d), and the units point. In parts (a) and (c) the student's work is correct. In part (b) the student does not mention the time interval, and the approximation is incorrect. In part (d) the student's work is incorrect. The student presents the correct units in part (b), so the units point was earned. The units in part (a) are incorrect.

Question 6



Graph of g

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

(a)
$$\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right)\right) dx$$
$$= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi}$$
$$= 6\pi^2$$

$2 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b)
$$f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$$

$f'(x)$ does not exist at $x = 0$.
For $-2\pi < x < 0$, $f'(x) \neq 0$.
For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

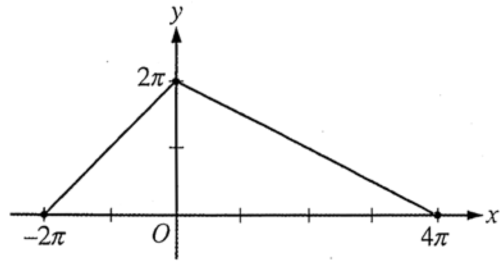
f has critical points at $x = 0$ and $x = \pi$.

$4 : \begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

(c)
$$h'(x) = g(3x) \cdot 3$$
$$h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED



Graph of g

Do not write beyond this border.

Work for problem 6(a)

$$f(x) = g(x) - \cos\left(\frac{x}{2}\right)$$

$$\int_{-2\pi}^{4\pi} f(x)\,dx$$

$$= \int_{-2\pi}^{4\pi} g(x) - \cos\left(\frac{x}{2}\right)\,dx$$

$$= \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\left(\frac{x}{2}\right)dx$$

$$= (4\pi+2\pi)\cdot 2\pi\cdot\frac{1}{2} - \int_{-2\pi}^{4\pi} \cos\left(\frac{x}{2}\right)dx$$

$$= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{-2\pi}^{4\pi}$$

$$= 6\pi^2 - \left[2\sin(2\pi) - 2\sin(-\pi)\right]$$

$$= 6\pi^2 - [0-0]$$

$$= 6\pi^2$$

Do not write beyond this border.

Continue problem 6 on page 15.

Work for problem 6(b)

$f'(x) = g'(x) - \cos'(\frac{x}{2})$

$= g'(x) + \frac{1}{2}\sin(\frac{x}{2})$

critical point: ① $f'(x)$ undefined at $x=0$

$f'(x) = 0.$ $g'(x) = -\frac{1}{2}\sin(\frac{x}{2})$

on $(-2\pi, 0)$ $\sin(\frac{x}{2}) = -2$. not exist.

on $(0, 4\pi)$ $\cancel{-\frac{1}{2}}\sin(\frac{x}{2}) = \frac{-2\pi}{4\pi} = \cancel{-\frac{1}{2}}$ $\sin(\frac{x}{2}) = 1$

$\frac{x}{2} = \frac{\pi}{2}, \frac{5}{2}\pi, \ldots$

② $x \in (0, 4\pi)$ $\therefore x = \pi$

Therefore. $x=0$, $x=\pi$. f has a critical point.

Work for problem 6(c)

$h(x) = \int_0^{3x} g(t)dt$

$h'(x) = 3g(3x)$

$h'(-\frac{\pi}{3}) = 3 \cdot g(-\pi)$

$= 3 \cdot \pi$

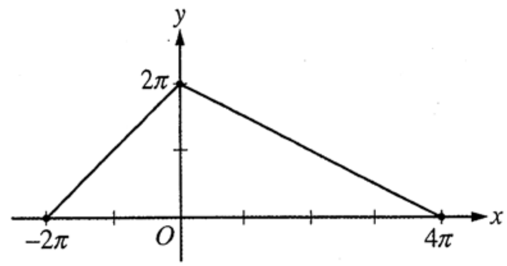
$h'(-\frac{\pi}{3}) = 3\pi.$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



Graph of g

Work for problem 6(a)

$$\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\tfrac{x}{2}\right)\right) dx$$

$$= \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\left(\tfrac{x}{2}\right) dx$$

$$= \int_{-2\pi}^{0} g(x)\,dx + \int_{0}^{4\pi} g(x)\,dx - 2\int_{-2\pi}^{4\pi} \tfrac{1}{2}\cos\left(\tfrac{x}{2}\right) dx$$

$$= \tfrac{1}{\cancel{2}}(\cancel{2}\pi)(2\pi) + \tfrac{1}{\cancel{2}}(4\pi)(\cancel{2}\pi) - \left[(2)\left[\sin \tfrac{x}{2}\Big|_{-2\pi}^{4\pi}\right.\right.$$

$$= 2\pi^2 + 4\pi^2 - \left[(2)\left[\sin 2\pi - \sin(-\pi)\right]\right]$$

$$= 6\pi^2 - \left[-2\sin(-\pi)\right]$$

$$= 6\pi^2 + 2\sin(-\pi) = 6\pi^2$$

Do not write beyond this border.

Do not write beyond this border.

Continue problem 6 on page 15.

Work for problem 6(b)

$f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right)$

$= g'(x) + \frac{1}{2}\sin\frac{x}{2}$

$f'(x) = 0 \Longrightarrow g'(x) = -\frac{1}{2}\sin\frac{x}{2}$

$\therefore x = 4\pi$

$\boxed{\therefore x = 4\pi}$ is a critical point

because $g'(4\pi) = 0$

and $-\frac{1}{2}\sin\frac{4\pi}{2} = -\frac{1}{2}\sin 2\pi = 0$

$\underline{f'(x) = 0}$

Work for problem 6(c)

$h(x) = \int_0^{3x} g(t)\,dt$

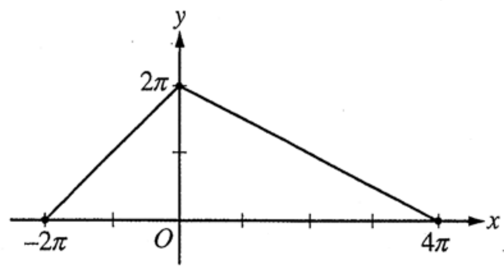
$\therefore h'(x) = 3g(3x)$

$h'\left(-\frac{\pi}{3}\right) = 3g\left(3\left(-\frac{\pi}{3}\right)\right)$

$= 3g(-\pi) = 3(\pi) = 3\pi$

Do not write beyond this border.

NO CALCULATOR ALLOWED



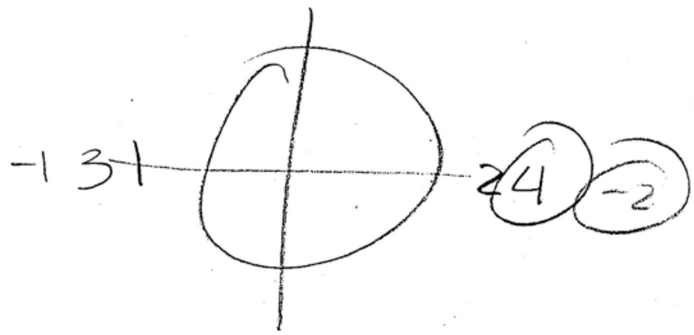
Graph of g

Do not write beyond this border.

Work for problem 6(a)

$f(x)=g(x)-\cos\left(\frac{x}{2}\right)$

$A=\int_{-2\pi}^{4\pi} g(x)-\cos\frac{x}{2}\,dx$



$A=\int_{-2\pi}^{4\pi} g(x)\,dx+\int_{-2\pi}^{4\pi}-\cos\frac{x}{2}\,dx$

$=\left(\cancel{\frac{1}{2}}\,2\pi(2\pi)+\cancel{\frac{1}{2}\,4\pi}(2\pi)\right)+$

$=\left(2\pi^2+4\pi^2\right)+$

$=6\pi^2+\int_{-2\pi}^{4\pi}-\cos\frac{x}{2}\,dx$

$= \quad \downarrow \quad +\frac{1}{2}\sin\frac{x}{2}\Big|_{-2\pi}^{4\pi}$

$= \quad \downarrow \quad +\frac{1}{2}(0-0)=6\pi^2$

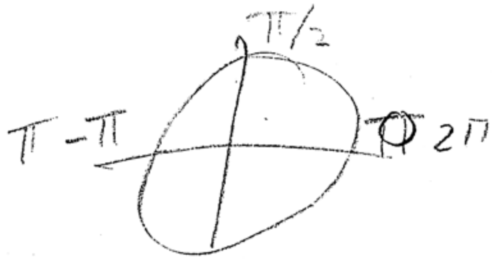
Continue problem 6 on page 15.

Work for problem 6(b)

$f(x) = g(x) - \cos\left(\frac{x}{2}\right)$

$0 = g(x) - \cos\left(\frac{x}{2}\right)$

$\cos\left(\frac{x}{2}\right) = g(x)$



?

Work for problem 6(c)

$h(c) = \int_0^{3x} g(t)\,dt$

$h'(c) = g(3x)$

$h'\left(-\frac{\pi}{3}\right) = g\left(-\frac{\pi}{\cancel{3}}(\cancel{3})\right)$

$= g(-\pi) = \pi$

$h'\left(-\frac{\pi}{3}\right) = \pi$

☆ fundamental theorem of calculus

Do not write beyond this border.

Do not write beyond this border.

Question 6

Sample: 6A
Score: 9

The student earned all 9 points. In part (b) the student earned the $g'(x)$ point implicitly on the fifth and sixth lines of the solution. The student presents multiple solutions on the seventh line but then reports the desired ones.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In parts (a) and (c) the student's work is correct. In part (b) the student earned the first point. The student does not present $g'(x)$ and does not identify the critical points.

Sample: 6
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), and 2 points in part (c). In part (a) the student has an antidifferentiation error on the next to last line, so the first point was not earned. The student was eligible for the answer point and earned it. In part (c) the response is missing a factor of 3 and so earned only 1 of the 2 points for $h'(x)$. The student was eligible for the answer point and earned it.